

L.M

JUN 18 1969

SHOCK TEMPERATURE CALCULATIONS  
FOR SILICONE FLUID\*

by

M. Cowperthwaite and J. H. Blackburn  
Stanford Research Institute  
Menlo Park, California

\* This work was supported by Physical Research Laboratory, Edgewood Arsenal, under Contract No. DA-18-035-AMC-122 (A).

---

Paper to be presented at Symposium on the Accurate Characterization of the High-Pressure Environment, October 14-18, 1968, U. S. Dept. of Commerce, National Bureau of Standards, Gaithersburg, Maryland.

## ABSTRACT

The problem of calculating shock temperature indirectly from experimental data without assuming thermodynamic properties is formulated and solved theoretically. In principle, the (e-p-v) and (T-p-v) equations of state can be constructed in an overlapping domain of the (p-v) plane from a family of Hugoniot curves centered at points of known energy and temperature.

Experiments were performed in an attempt to construct these equations of state for silicone fluid 210. Shock and free surface velocities were measured to determine Hugoniot curves in the 300-kbar regime, and energies and densities were measured from  $-30^{\circ}$  to  $+260^{\circ}\text{C}$  along the atmospheric isobar to determine the initial states of the shock wave experiments. In practice, it was necessary to assume a form for the (e-p-v) equation of state, since the differences in volumes between states on Hugoniot curves at the same pressure above 40 kbar were found to be comparable with the experimental error in measuring the volumes of each of these states. The data were fitted to a Mie-Grüneisen type (e-p-v) equation of state with variable  $C_v$  and  $(\partial p/\partial T)_v$ , since Hugoniot points indicated a linear dependence of energy on pressure along an isochore, and  $C_v$  varied along the atmospheric isobar. Shock temperatures on the  $25^{\circ}\text{C}$  Hugoniot were calculated at points of intersection with isentropes and by integrating with constant atmospheric pressure values of  $C_v$ . The position of the  $296^{\circ}\text{C}$  isentrope limits the temperature calculation with isentropes to values below  $522^{\circ}\text{C}$  and 58 kbar, and the values around 50 kbar are 8% lower than those calculated with the  $25^{\circ}\text{C}$  value of  $C_v$ , since  $C_v$  increases along the Hugoniot curve. Temperature calculations above 58 kbar assumed the  $296^{\circ}\text{C}$  value of  $C_v$ . Considerably more experimental work would be required over the entire pressure range to permit determining equations of state of silicone 210 without making assumptions.

## II. THEORY

We first pose the problem of shock temperature and formulate a theoretical basis for its solution. Let  $e$ ,  $s$ ,  $U$ , and  $u$  denote specific energy, specific entropy, shock velocity, and particle velocity, and let subscript  $o$  denote the constant state of stationary fluid in front of the shock. Then the Rankine-Hugoniot jump<sup>6</sup> conditions relating shocked and unshocked states,

$$vU = v_o(U - u) \quad (1)$$

$$uU = v_o(p - p_o) \quad (2)$$

$$p v_o = U(e - e_o + \frac{1}{2}u^2) \quad (3)$$

express the balance of mass, momentum, and energy across the shock discontinuity, and the inequality

$$s(e, v) > s(e_o, v_o)$$

expresses the second law of thermodynamics for the irreversible shock process.

Eliminating  $U$  and  $u$  from Eq. 3 gives the Hugoniot equation<sup>7</sup>

$$e - e_o = \frac{1}{2}(p + p_o)(v_o - v). \quad (4)$$

If an  $(e-p-v)$  equation of state satisfies the condition  $(\partial^2 p / \partial v^2)_s > 0$ , then Eq. 4 with  $v < v_o$  defines the locus of compressed states on the  $(e-p-v)$  surface that can be reached from an initial condition  $(e_o, p_o, v_o)$  by single shocks. The  $(e-p-v)$  equation of state and Eq. 4 define this locus of shocked states as a curve in the  $(p, v)$  plane,  $p = p_H(p_o, v_o, v)$ , which passes through the point  $(p_o, v_o)$  and is called the Hugoniot curve centered at  $(p_o, v_o)$ . The elimination of  $u$  from Eqs. 1 and 2 gives the equation of the Rayleigh line,

$$p - p_o = (U/v_o)^2 (v_o - v). \quad (5)$$

Since a shocked state satisfies Eqs. 4 and 5, the intersection of the Hugoniot curve centered on  $(p_o, v_o)$  and the Rayleigh line of slope  $-(U/v_o)^2$  passing through  $(p_o, v_o)$  defines the mechanical thermodynamic state  $(p, v)$  behind a shock propagating at constant velocity  $U$  into a stationary state  $(p_o, v_o)$ .

With the assumption of thermodynamic equilibrium behind a shock, the state variables of a nonreacting shocked fluid satisfy the following thermodynamic identities:

$$de = Tds - pdv \quad (6)$$

$$T(s, v) = \left( \frac{\partial e}{\partial s} \right)_v \quad (7)$$

$$p(s, v) = - \left( \frac{\partial e}{\partial v} \right)_s \quad (8)$$

For thermomechanical processes, a knowledge of  $e$ ,  $s$ ,  $T$ ,  $p$ , and  $v$  provides a complete characterization of a thermodynamic state. Thus, the  $(e-s-v)$  equation of state is called complete because of the identities 7 and 8 that define the  $(T-s-v)$  and  $(p-s-v)$  equations of state, but all other equations of state among these variables are incomplete. The  $(e-p-v)$  equation of state is incomplete because it cannot be used to calculate temperature and entropy without additional data. Similarly, the  $(T-p-v)$  equation of state is incomplete because it cannot be used to calculate energy and entropy without additional data. However, a knowledge of any two incomplete equations of state provides a complete characterization because of the identities of thermodynamics.

The objective of the present work is to use shock wave and low pressure data to characterize completely the high pressure environment in the kilobar regime without additional thermodynamic assumptions. Since shock temperature cannot be measured directly with present-day techniques and cannot be calculated from knowledge of the energy along a Hugoniot curve, it is necessary to construct the  $(T-p-v)$  equation of state. Such a construction must be based on the mechanical properties of shocked states. At present the only feasible way to achieve this objective is to construct the  $(e-p-v)$  equation of state first, and then use it with the identities of thermodynamics to calculate the  $(T-p-v)$  relationship. Hugoniot curves form the basis of the experimental method of constructing the  $(e-p-v)$  equation of state using shock wave data; the relationship between the  $(T-p-v)$  and  $(e-p-v)$  equations of state forms the basis for calculating the temperature of shocked states.



## The (e-p-v) Equation of State

An experimental Hugoniot curve  $p_H(p_o, v_o, v)$  is the locus of experimentally measured pressure-volume states produced by passing constant velocity shocks of various strengths into an initial state  $(p_o, v_o)$ . The change in internal energy along a Hugoniot curve is given by Eq. 4. A family of Hugoniot curves, each of which is centered on a curve along which the energy change is known, is therefore sufficient to determine the (e-p-v) equation of state over the domain of the (p-v) plane covered by the Hugoniots.

In the present work we choose to measure a family of Hugoniots centered on the atmospheric ( $p \approx 0$ ) isobar, because the energy change can easily be measured along this cross curve.

## Calculation of the (T-p-v) from the (e-p-v) Equation of State

Since the (T-p-v) and (e-p-v) equations of state are both incomplete, it is necessary to establish what additional data are required to calculate temperature when the (e-p-v) relationship is known. It follows from thermodynamic identities that the (e-p-v) and (T-p-v) equations of state are related through isentropes. The position of an isentrope in the (p-v) plane is determined by the (e-p-v) equation of state and the isentropic condition  $de = -pdv$  obtained by setting  $ds = 0$  in Eq. 6. The temperature along an isentrope is given as

$$T = T_1 \exp \left[ - \int_{v_1}^v \left( \frac{\partial p}{\partial e} \right)_v dv \right] \quad (9)$$

by integrating the identity

$$ds = \left( \frac{\partial e}{\partial T} \right)_v \frac{dT}{T} + \left( \frac{\partial p}{\partial T} \right)_v dv \quad (10)$$

subject to the isentropic condition  $ds = 0$ .

Thus to calculate the (T-p-v) from the (e-p-v) equation of state, it is necessary first to construct a family of isentropes in the (p-v) plane and then to calculate temperature along them with Eq. 9. It is important to note that temperature cannot be calculated with Eq. 9 unless the temperatures  $T_1$  at particular points  $(p_1, v_1)$  on the isentropes are known. Measurement of the temperature along any curve which intersects the entire family of isentropes permits a value of  $T_1$  to be assigned to each isentrope. Thus the (T-p-v) equation of state is determined in the domain of the (p-v) plane covered by a family of isentropes. For a given (e-p-v) equation of state, there is a thermodynamically consistent (T-p-v) equation of state for each assignment of temperature along a nonisentropic curve.

Measurements of temperature and energy along the atmospheric isobar are sufficient to calculate the (T-p-v) and (e-p-v) equations of state from a family of Hugoniot curves centered on this isobar. However, the (e-p-v) and (T-p-v) equations of state will necessarily be specified over different but overlapping domains of the (p-v) plane. The family of Hugoniot curves defines the domain where the (e-p-v) equation of state is known, but the family of isentropes constructed from the (e-p-v) relationship defines the subdomain where the (T-p-v) equation of state is known.

### III. EXPERIMENTS

Dow Corning silicone 210 fluid (100 centistokes) was chosen because of its good thermal stability and its large coefficient of expansion. Static experiments were performed to measure the variation of density and specific enthalpy  $h^*$ , along the atmospheric isobar. Shock wave experiments were performed to determine a family of Hugoniot curves centered on the atmospheric isobar.

#### Static Measurements

The variation of volume with temperature at atmospheric pressure between  $-30^\circ\text{C}$  and  $150^\circ\text{C}$  was measured with a density balance. A least squares fit for the data, with T in degrees Kelvin,

$$v^{-1} = 1.2566 + 1.0577 \times 10^{-3}T + 2.604 \times 10^{-7}T^2$$

was used above 150°C to calculate initial conditions for the shock wave experiments. The standard deviation from the least squares fit is less than 0.1% of the largest volume measured.

The variation of specific enthalpy with temperature at atmospheric pressure between -26°C and 318°C was determined with a drop calorimeter. A least squares fit for the data with T in degrees Kelvin and h = 0 at  $T_1 = 298^\circ\text{K}$  is

$$h = 16960(1/T - 1/T_1) - 0.09818(T - T_1) + 4.842 \times 10^{-4}(T^2 - T_1^2)$$

The standard deviation from the least squares fit is less than 3/4% of the largest enthalpy increment measured.

#### Shock Measurements

Four explosive shots were performed to obtain high pressure equation of state data using an impedance match technique.<sup>1</sup> Each shot assembly contained both cold and hot liquid samples.

A cross section of an assembly showing the brass cells containing the liquid samples is illustrated in Fig. 1. Plane shocks were induced in the liquid samples by the interactions produced by a brass flier striking the cells. The driver system for the flier plate was a P-80 plane-wave lens in contact with a 4-inch pad of high explosive. Shocked conditions in the liquid were varied by varying the composition of the explosive pad.

Direct measurement of shock velocity in the liquid and indirect measurement of the shocked condition in the brass at the cell-liquid interface suffice to calculate the shocked state in the liquid. The measurements were recorded on 70-mm Tri-X film with a Beckmann & Whitley 770 camera writing at a speed of 10 mm/μsec; object-to-image ratio was 2.6/1 and the slit overwrite time was 0.01 μsec. Figure 2 shows a detailed drawing of a streak camera view of the liquid cells. Changes of reflectivity of the steel shims E and C, the brass surface DF, and the mirrored surface H produce signals that depict the series of events for the cold cell in the shock experiment. A typical streak record is shown in Fig. 3,



where the letters identifying the signals match the letters in Fig. 2. The figure shows (a) the flier plate striking the cold cell at C and J; (b) the time of arrival of the shock at the surface of the brass at D, F, G, and I; (c) the time of arrival of the free surface of the brass at E; (d) entrance of the shock into the liquid at G and I; and (e) time of arrival of the shock at the liquid-glass interface at H. The shock velocity through the liquid was calculated from times recorded at G, I, and H. The shock velocity in the cold brass and its free surface velocity determine conditions at the brass-liquid interface. The shock velocity in the cold cell was calculated using the time differences of location C and D divided into a corrected thickness of the brass at location D. The corresponding free-surface velocity was calculated from times at D, F, and E. The time differences between locations were reduced by 0.022  $\mu$ sec to account for the transit time through the steel shim at location E. Similar calculations were made for the hot cell.

Pressure in the brass was calculated using Eq. 2 with  $p_o = 0$  and with the particle velocity assumed to be one-half free surface velocity. Comparison of the calculated (p-u) points for hot and cold brass with the (p-u) points of McQueen and Marsh<sup>8</sup> for brass initially at 20°C shows that only the 20°C Hugoniot curve need be considered for the impedance match calculations. The mirror-image approximation was used for the brass isentropes. Thus pressure and particle velocity at the brass-liquid interface were calculated at the intersection of the Rayleigh line for the liquid and the brass rarefaction curve, assumed to be a reflection of the Hugoniot curve through the measured free surface velocity. The specific volume of the shocked liquid was then calculated using Eq. 1, and the corresponding change of internal energy was calculated using the Hugoniot equation in the form  $e - e_o = \frac{1}{2}u^2$ .

The shock wave data are summarized in Table I. The precision of the data is controlled mainly by the pressure uniformity and planarity of the explosive driver system.



#### IV. CONSTRUCTION OF (e-p-v) EQUATION OF STATE

The limited number of experimental Hugoniot points and the restricted range of data along the atmospheric isobar prohibit the construction of an equation of state solely from experimental data. It is important, however, to use both shock wave and static data to indicate the most appropriate form of the (e-p-v) equation of state. A graphic fit of Hugoniot data in the 200-kbar regime, without the three points from shots 12,326 and 12,496 that indicate crossing Hugoniot curves, suggests a linear dependence of internal energy on pressure along lines of constant volume-isochores; static data show that the partial derivative  $(\partial e / \partial p)_v$  varies along the atmospheric isobar. Thus, the (e-p-v) data were fitted to the form

$$e = pf(v) + g(v) \quad (11)$$

with  $(\partial e / \partial p)_v = f(v) > 0$  everywhere in the region of interest.

Additional properties of this model follow from thermodynamic relationships. The relationship between specific heat at constant pressure  $C_p$  and specific heat at constant volume  $C_v$  is

$$C_p = C_v [1 + T(\partial v / \partial T)_p / f(v)] \quad (12)$$

and  $C_v$  is constant along an isentrope, since

$$\left( \frac{\partial C_v}{\partial T} \right)_s = \left( \frac{\partial^2 e}{\partial p^2} \right)_v \left( \frac{\partial p}{\partial T} \right)_v^2 = 0 \quad (13)$$

The equation for a Hugoniot curve centered at  $(p_0 = 0, v_0)$  is

$$p[f(v) - \frac{1}{2}(v_0 - v)] = g(v_0) - g(v) \quad (14)$$

the differential equation for an isentrope is

$$\left( \frac{\partial p}{\partial v} \right)_s = - \frac{p(1 + df/dv) + dg/dv}{f(v)} \quad (15)$$

and the equation<sup>9</sup> obtained by formal integration of Eq. 15 shows that the first derivative of  $g(v)$  must be positive, i.e.,  $dg/dv > 0$ . The rapid increase of pressure along an isentrope indicates that the (e-p-v) relationship will satisfy the mechanical stability condition  $(\partial p / \partial v)_s < 0$  if  $f(v)$  satisfies the condition  $(1 + df/dv) > 0$ .

Values of  $f(v)$  in the specific volume range  $0.994 < v < 1.35$  cc/g along the atmospheric isobar were calculated with the identity,

$$f(v) = -C_p \left( \frac{\partial v}{\partial p} \right)_s \left( \frac{\partial T}{\partial v} \right)_p \quad (16)$$

The values of  $C_p$  and  $(\partial v / \partial T)_p$  were determined experimentally. The values of  $(\partial p / \partial v)_s$  were calculated from the sound velocity data of McSkimin<sup>10</sup>, which were extrapolated to cover the range of initial temperatures used in the shock experiments. Values of  $f(v)$  in the volume range  $0.515 < v < 0.55$  cc/g were taken to be the slopes of (e-p) isochores calculated from the Hugoniot curves. Values of  $f(v)$  in the intermediate range were assumed to lie on a smooth curve because values of  $(\partial e / \partial p)_v$  calculated in the neighborhood of 0.54 cc/g were approximately equal to the value calculated at 0.994 cc/g. Least squares fits of the data give the following expressions for  $f(v)$ :

$$\begin{aligned} f(v) &= -23.055 + 23.134v & \text{if } v \geq 1.152 \text{ cc/g} \\ f(v) &= 60.502 - 121.866v + 62.916v^2 & \text{if } 0.9693 \leq v \leq 1.152 \text{ cc/g} \\ f(v) &= 1.3822 + 0.108v & \text{if } v \leq 0.9693 \text{ cc/g} \end{aligned}$$

where the constants are given to a number of decimal places for computation.

Since  $h = e = g(v)$  when  $p = 0$ , the measured enthalpies at atmospheric pressure give values of  $g(v)$  in the volume range  $0.985 \leq v \leq 1.66$  cc/g. A linear least squares fit for  $g(v)$  in this volume range is given by the expression

$$g(v) = -16.107 + 15.517v.$$

For values of volume less than 0.985 cc/g, fits for  $g(v)$  were generated by patching together the high pressure Hugoniot data and the atmospheric data so as to satisfy the condition  $dg/dv > 0$ . The best least squares fits for  $g(v)$  with a slight discontinuity in the slope at  $v = 1.01316$  cc/g are:

$$\begin{aligned} g(v) &= 2408.116 + 7566.432v - 7949.11v^2 + 2787.845v^3 & \text{if } v \leq 1.0136 \text{ cc/g} \\ g(v) &= -16.107 + 15.517v & \text{if } v \geq 1.0136 \text{ cc/g} \end{aligned}$$

The fits for  $f(v)$  and  $g(v)$  define the  $(e-p-v)$  equation of state and allow calculation of Hugoniot curves centered on, and isentropes passing through points on the  $p=0$  isobar in the  $(p-v)$  plane. The discontinuous change in slope of the  $g(v)$  function at  $v = 1.01316$  cc/g is manifest in the shape of these curves in the neighborhood of this volume.

## V. CALCULATIONS

### Construction of Hugoniot Curves and Isentropes

The Hugoniot curves centered on  $p = 0$  at  $-20^\circ\text{C}$ ,  $25^\circ\text{C}$ ,  $158.5^\circ\text{C}$ , and  $256^\circ\text{C}$  were calculated directly with Eq. 14. The isentropes passing through  $p = 0$  at  $25^\circ\text{C}$ ,  $158.5^\circ\text{C}$ ,  $256^\circ\text{C}$ , and  $296^\circ\text{C}$  were constructed by integrating Eq. 15 numerically with a Runge-Kutta technique. The  $-20^\circ\text{C}$ ,  $158.5^\circ\text{C}$ , and  $256^\circ\text{C}$  Hugoniot curves and the  $296^\circ\text{C}$  isentrope are shown in Fig. 4. The  $25^\circ\text{C}$  Hugoniot and the  $25^\circ\text{C}$  and  $296^\circ\text{C}$  isentropes are shown in Fig. 5.

### Calculation of Temperature

Equation 9 was used to calculate the temperature along the isentropes passing through  $25^\circ\text{C}$  and  $296^\circ\text{C}$  on the atmospheric isobar. The values of temperature along these isentropes are listed in Table II. Calculation of temperature at points where isentropes intersect the  $25^\circ\text{C}$  Hugoniot defines values of shock temperature along this Hugoniot curve. The point of intersection ( $T_c = 522.1^\circ\text{C}$ ,  $p_c = 58$  kbar,  $v_c = 0.661$  cc/g) of the  $296^\circ\text{C}$  isentrope and the  $25^\circ\text{C}$  Hugoniot is the highest point on the  $25^\circ\text{C}$  Hugoniot where shock temperature can be calculated with the present data. The temperature on the  $25^\circ\text{C}$  Hugoniot below 58 kbar can be calculated with the isentropes lying to the left of the  $296^\circ\text{C}$  isentrope. The temperature where the  $256^\circ\text{C}$  isentrope intersects the Hugoniot is  $456.7^\circ\text{C}$ , and the temperature where the  $158.5^\circ\text{C}$  isentrope intersects the Hugoniot is  $291.5^\circ\text{C}$ .

It is not possible to calculate temperature on the  $25^\circ\text{C}$  Hugoniot above 58 kbar without making further assumptions. The temperature along the Hugoniot above 58 kbar was calculated with constant  $C_v$  rather than by extrapolating the low pressure data further. The equation for shock temperature<sup>2</sup> above a point  $(T_c, v_c)$  on a Hugoniot centered at  $(p_0 = 0, v_0)$ ,



$$T_I = T_c + \frac{1}{2C_v} \int_{v_c}^v I \left[ p + (v_o - v) \frac{dp}{dv_H} \right] dv \quad (17)$$

with

$$I = \exp \int_{v_c}^v dv/f(v)$$

was used to calculate temperature on the 25°C Hugoniot above ( $T_c = 522.1^\circ\text{C}$ ,  $v_c = 0.661 \text{ cc/g}$ ). The integral term in Eq. 17 was evaluated numerically with a constant value of  $C_v = 1.3735 \times 10^{-2} \text{ kbar cm}^3/\text{g}^\circ\text{C}$  equal to the constant value along the 296°C isentrope. Since  $C_v$  is constant along an isentrope, the significance of temperatures calculated with Eq. 17 depends on the variation of  $C_v$  with volume along the atmospheric isobar above  $v = 1.35 \text{ cc/g}$ . In the case that  $C_v$  increases with increasing volume above  $v = 1.35 \text{ cc/g}$ , it also increases with the increasing pressure along the Hugoniot curve, and the values of temperature calculated with Eq. 17 under the assumption of constant  $C_v$  would be upper estimates for shock temperature above 58 kbar. The values of temperature calculated with Eq. 17 are listed in Table III.

For comparison the method of Walsh and Christian<sup>2</sup> was also used to calculate temperature along the 25°C Hugoniot curve with Eq. 17. The integral was evaluated under the assumption that  $C_v$  and  $f(v)$  were constant along the Hugoniot curve and that the values of these constants were the values of  $C_v$  and  $f(v)$  evaluated at 25°C. The calculated temperatures where the 158.5°C, 256°C, and 296°C isentropes intersect the 25°C Hugoniot are 296.8°C, 507.1°C, and 561.5°C. The values of temperature above 58 kbar calculated by this method are also listed in Table III.

## VI. SUMMARY AND CONCLUSIONS

Because of the scarcity and inaccuracy of experimental data, it was necessary to assume a simple form of the (e-p-v) equation of state to calculate the thermodynamic properties of silicone fluid. The particular form of the (e-p-v) relationship,  $e = pf(v) + g(v)$  with  $f(v)$  and  $g(v)$  arbitrary functions of volume, was suggested from the shock wave data and also from the variation of  $(\partial e/\partial p)_v$  along the atmospheric isobar. Values of  $f(v)$  calculated from the shock wave data and values of  $f(v)$  calculated from atmospheric static data with the identity  $(\partial e/\partial p)_v = -C_p (\partial v/\partial p)_s (\partial T/\partial v)_p$



were used to calculate values of  $f(v)$  in the volume range not covered by the experimental data. The values of  $g(v)$  calculated from shock wave data, the values of  $g(v)$  measured along the atmospheric isobar, and the condition  $dg/dv > 0$  were used to calculate values of  $g(v)$  in the volume range not covered by experiment.

The Hugoniot curves centered at  $-20^{\circ}\text{C}$ ,  $25^{\circ}\text{C}$ ,  $158.5^{\circ}\text{C}$ , and  $256^{\circ}\text{C}$  on the atmospheric isobar were calculated directly with the expression  $p[f(v) - \frac{1}{2}(v_0 - v)] = g(v_0) - g(v)$  obtained by combining the Hugoniot equation with the (e-p-v) equation of state. The calculated Hugoniot curves were consistent with experimental Hugoniot points up to a pressure of 240 kbar but then started to deviate from them. The isentropes passing through  $25^{\circ}\text{C}$ ,  $158.5^{\circ}\text{C}$ ,  $256^{\circ}\text{C}$ , and  $296^{\circ}\text{C}$  on the atmospheric isobar were calculated by numerically integrating the differential equation for an isentrope with a Runge-Kutta method; the temperature along these isentropes was calculated with the equation  $T = T_1 \exp - \int_{v_1}^v dv/f(v)$ . Calculation of the temperature where the  $158.5^{\circ}\text{C}$ ,  $256^{\circ}\text{C}$ , and  $296^{\circ}\text{C}$  isentropes intersect the  $25^{\circ}\text{C}$  Hugoniot curve defines values of shock temperature on this Hugoniot curve. The temperature  $T_c = 522.1^{\circ}\text{C}$  at the point of intersection ( $p_c = 58$  kbar,  $v_c = 0.661$  cc/g) of the  $296^{\circ}\text{C}$  isentrope and the  $25^{\circ}\text{C}$  Hugoniot is the highest temperature on the  $25^{\circ}\text{C}$  Hugoniot that can be calculated from the data without additional assumptions. To put a possible upper estimate on shock temperature along the  $25^{\circ}\text{C}$  Hugoniot above 58 kbar, the temperature was calculated by integrating along the Hugoniot curve with constant  $C_v$ . The method of Walsh and Christian was used to calculate temperature along the  $25^{\circ}\text{C}$  Hugoniot above 0 kbar.

It is obvious from our calculations of the (e-p-v) equation of state that many more experimental Hugoniot (p-v) points are needed to construct an (e-p-v) equation of state without first assuming its functional form. Indeed, to test the feasibility of constructing an equation of state from experimental data it would be necessary to determine, with accuracy, the positions of at least three Hugoniot curves in the (p-v) plane. With well-defined Hugoniot curves it is possible to test the validity of thermodynamic assumptions such as  $(\partial e/\partial p)_v = f(v)$ , and if necessary to

fit the data with more complicated functions. However, with a limited amount of Hugoniot data and the positions of the Hugoniot curves not well defined, it is not in general possible to test or to determine any thermodynamic properties conclusively. Specifically, it is not possible to test the validity of  $(\partial e / \partial p)_v = f(v)$  with our data because the experimental Hugoniot points do not lie on isochores in the (p-v) plane. Many more data points obtained with explosive driver systems producing an improvement in wave flatness and pressure uniformity are required to achieve a more definitive characterization of the (e-p-v) surface.

The temperature calculations in this paper use more thermodynamic data than previous methods of calculating shock temperature. The shock wave Hugoniot data and the atmospheric data for silicone fluid span a larger domain of the (p-v) plane than either the water data used by Rice and Walsh<sup>3</sup> or the metals data used by Walsh and Christian.<sup>2</sup> In the domain of the (p-v) plane where it is possible to calculate temperature without making assumptions about specific heat, the present method of calculating temperature is considered to be better than methods based on the assumption of a constant specific heat. The comparison of temperatures on the 25°C Hugoniot curve below 58 kbar calculated using the present method, with those calculated using the Walsh and Christian model based on the constancy of  $C_v$  and  $(\partial e / \partial p)_v$  suggests that the Walsh and Christian model gives an upper estimate for shock temperature. The comparison of temperatures on this Hugoniot above 58 kbar calculated using the Walsh and Christian method but with different values of  $C_v$  substantiates this suggestion. However, the difference between the temperatures calculated above 58 kbar resulting from the difference between the numerical values of the constants emphasizes the problem of determining the temperature of shocked liquids. The fact that it is possible with the Walsh and Christian model to calculate different temperatures with a constant value of  $(\partial e / \partial p)_v$  once again reflects the independence of the (e-p-v) and (T-p-v) equations of state. Because of the identity  $(\partial e / \partial p)_v = C_v / (\partial p / \partial T)_v$ , the (T-p-v) equation of state is not determined by an (e-p-v) equation of state based on an experimental Hugoniot curve and the assumption of constant  $(\partial e / \partial p)_v$ . The constancy

of  $(\partial e / \partial p)_v$  provides no guarantee that  $C_v$  and  $(\partial p / \partial T)_v$  are also constant. In fact different sets of values of  $C_v$  and  $(\partial p / \partial T)_v$  that satisfy the condition of constant  $(\partial e / \partial p)_v$  can be associated with different (T-p-v) equations of state that will give different values of shock temperature.

It is concluded that the values of shock temperature calculated along the isentropes are more realistic than those that would be calculated with models based on less thermodynamic information.

#### ACKNOWLEDGMENTS

The authors are grateful to Physical Research Laboratory, Edgewood Arsenal, for supporting this work under Contract DA-18-035-AMC-122(A), and thank Miss B. Loo for performing the numerical integration and plotting the diagrams.



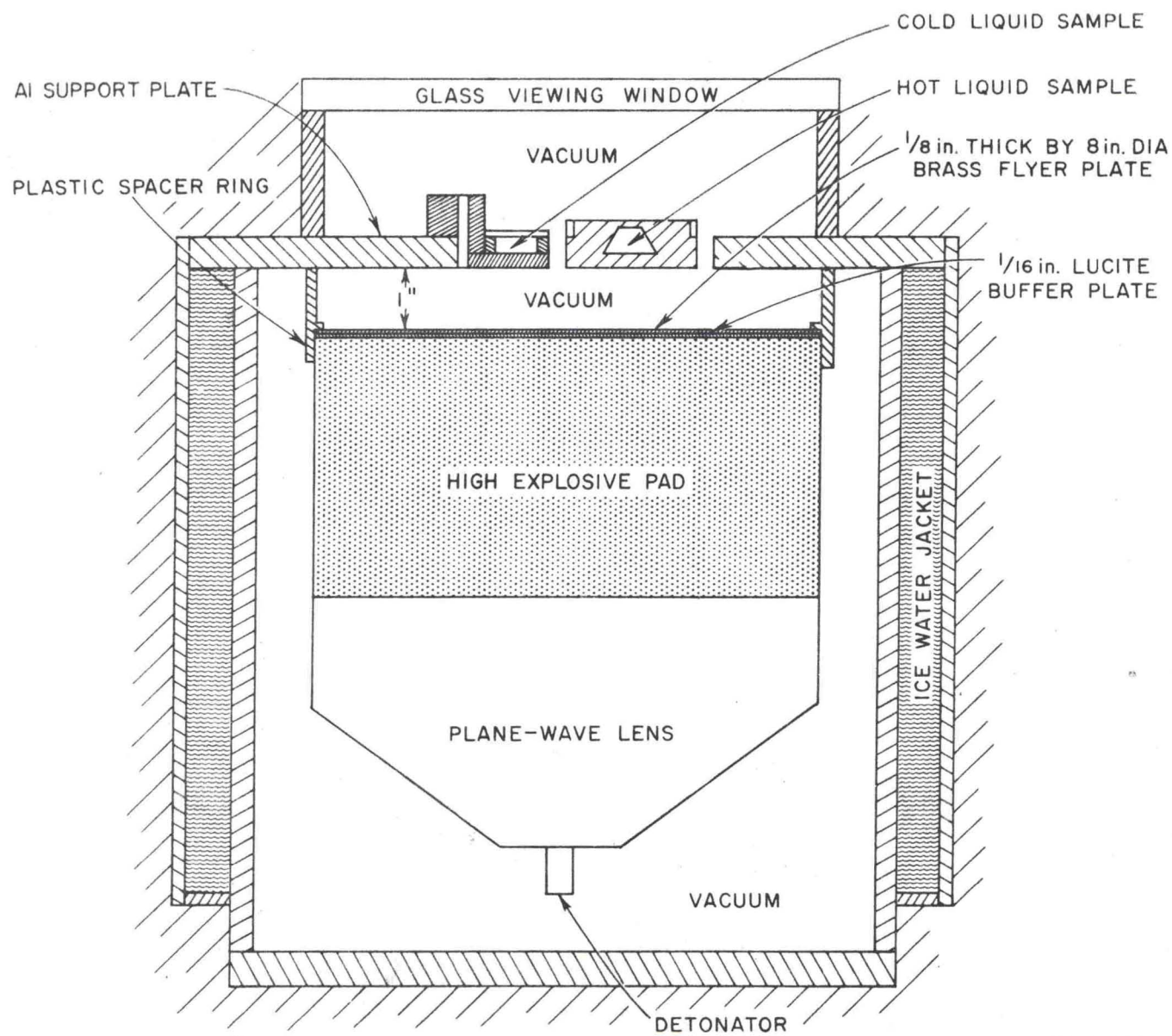
## REFERENCES

1. M. H. Rice, R. G. McQueen, and J. M. Walsh, "Compression of Solids by Strong Shock Waves," Solid State Physics, Vol. 6 (Academic Press, Inc., 1958).
2. J. M. Walsh and R. H. Christian, Phys. Rev. 97 (1544), 1955.
3. M. H. Rice and J. M. Walsh, J. Chem. Phys. 26 (824), 1957.
4. M. Cowperthwaite, Am. J. Phys. 34 (1025), 1966.
5. J. W. Gibbs, The Scientific Papers of J. Willard Gibbs; Vol. 1, Thermodynamics, Dover Publications, New York, 1960.
6. Ya B. Zeldovich and A. S. Kompaneets, Theory of Detonation, Academic Press, New York & London, 1960, pp 9-10.
7. R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves, Interscience Publishers, New York, 1948.
8. R. G. McQueen and S. P. Marsh, "Equation of State for 19 Metallic Elements and Shock Wave Measurements to 2 Megabars," J. Appl. Phys., 31, 7, 1253-1269, 1960.
9. M. Cowperthwaite and J. H. Blackburn, "Temperature of Shocked Liquids," Stanford Research Institute Special Technical Report No. 14, Contract DA-18-035-AMC-122(A), 1967.
10. H. J. McSkimin, J. Acoust. Soc Am. 29, 1185, 1957.



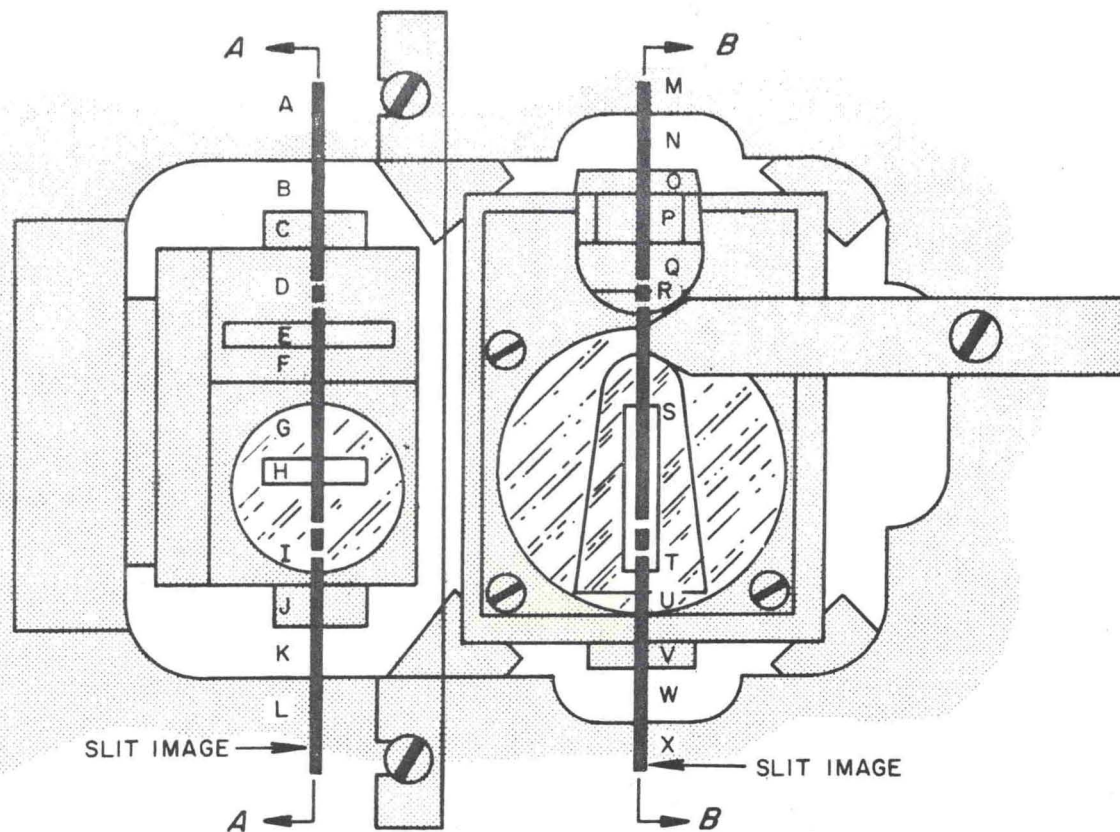
## FIGURE CAPTIONS

1. Cross-Section of Shock Experiment
2. Hot and Cold Cells: (a) Camera view of hot and cold cells showing slit projection; (b) Cross-sectional views through hot and cold cells along slit lines.
3. Streak Camera Photographs: (a) Static (b) Dynamic
4. Isentrope and Three Hugoniot Calculated for Silicone 210
5. Hugoniot and Two Isentropes Calculated for Silicone 210



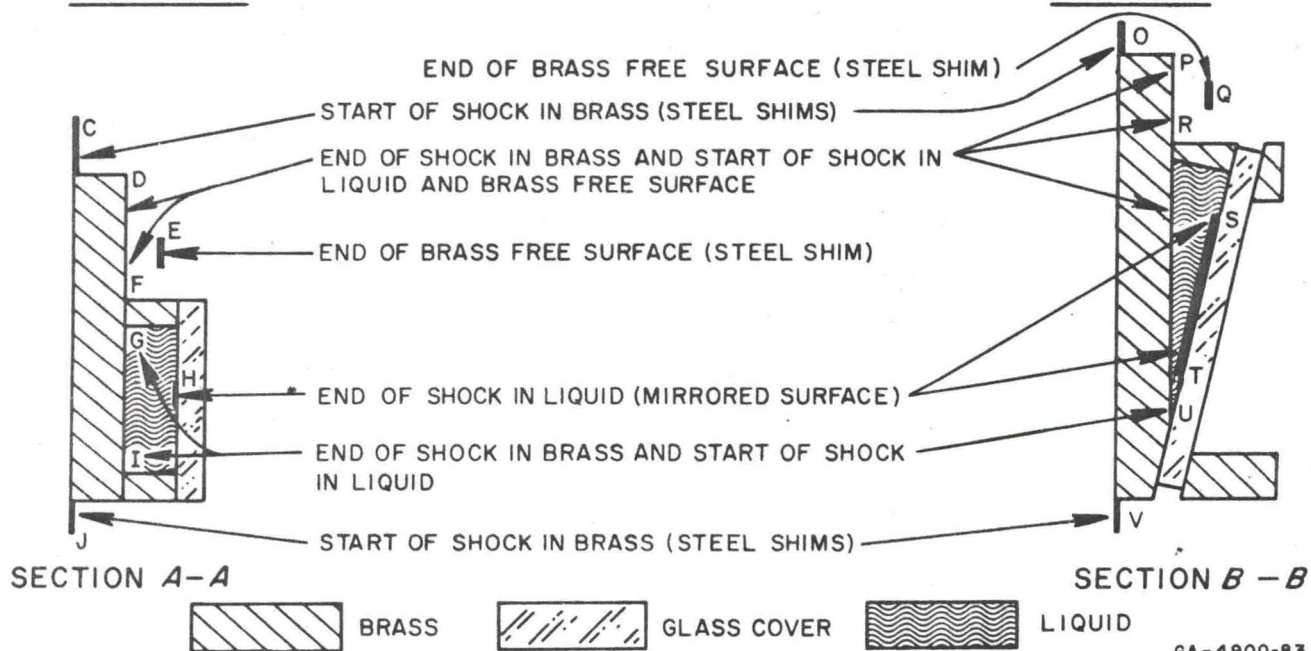
TB-4900-90

Fig. 1



# COLD CELL

# HOT CELL



GA-4900-83

Fig. 2



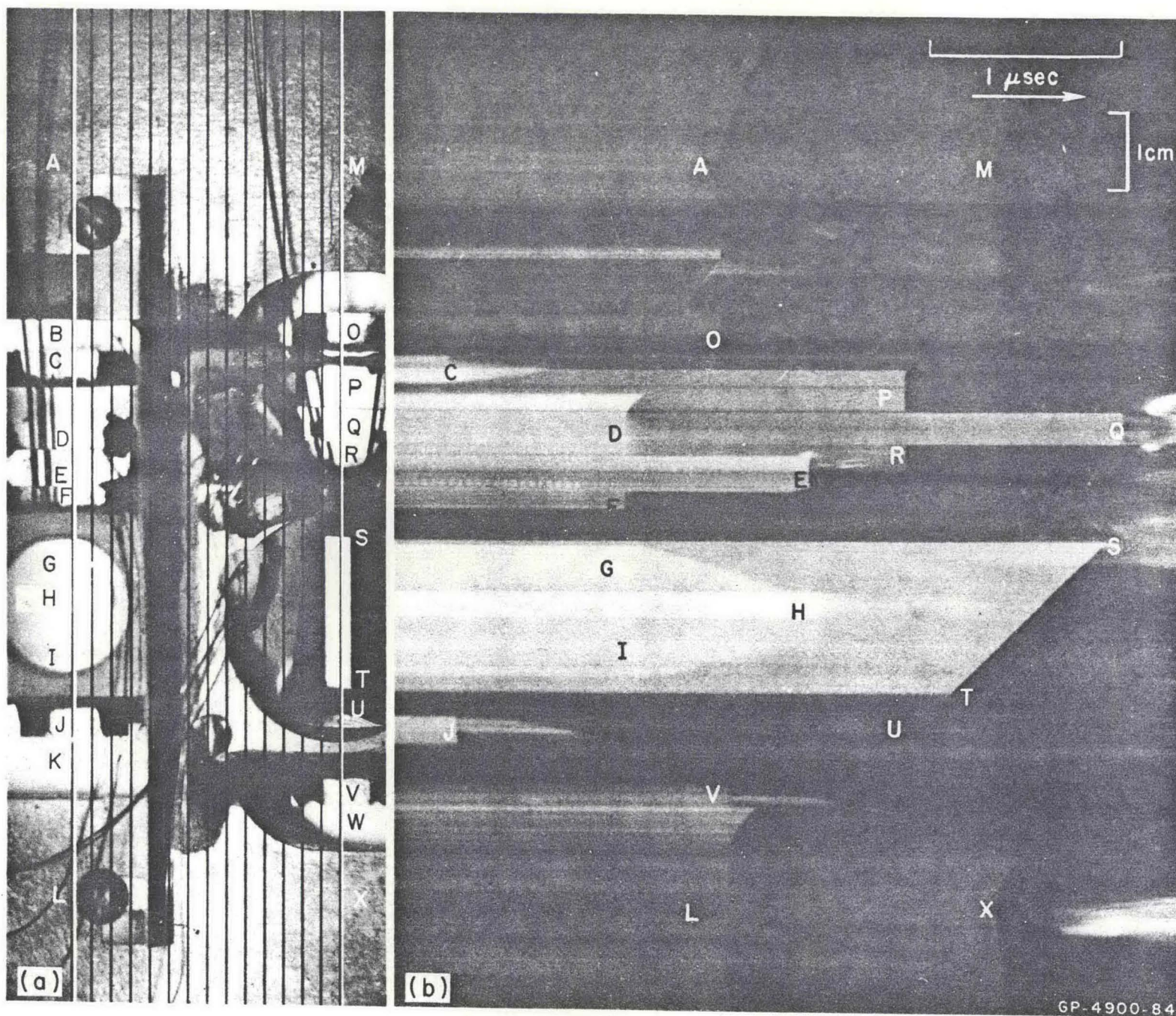
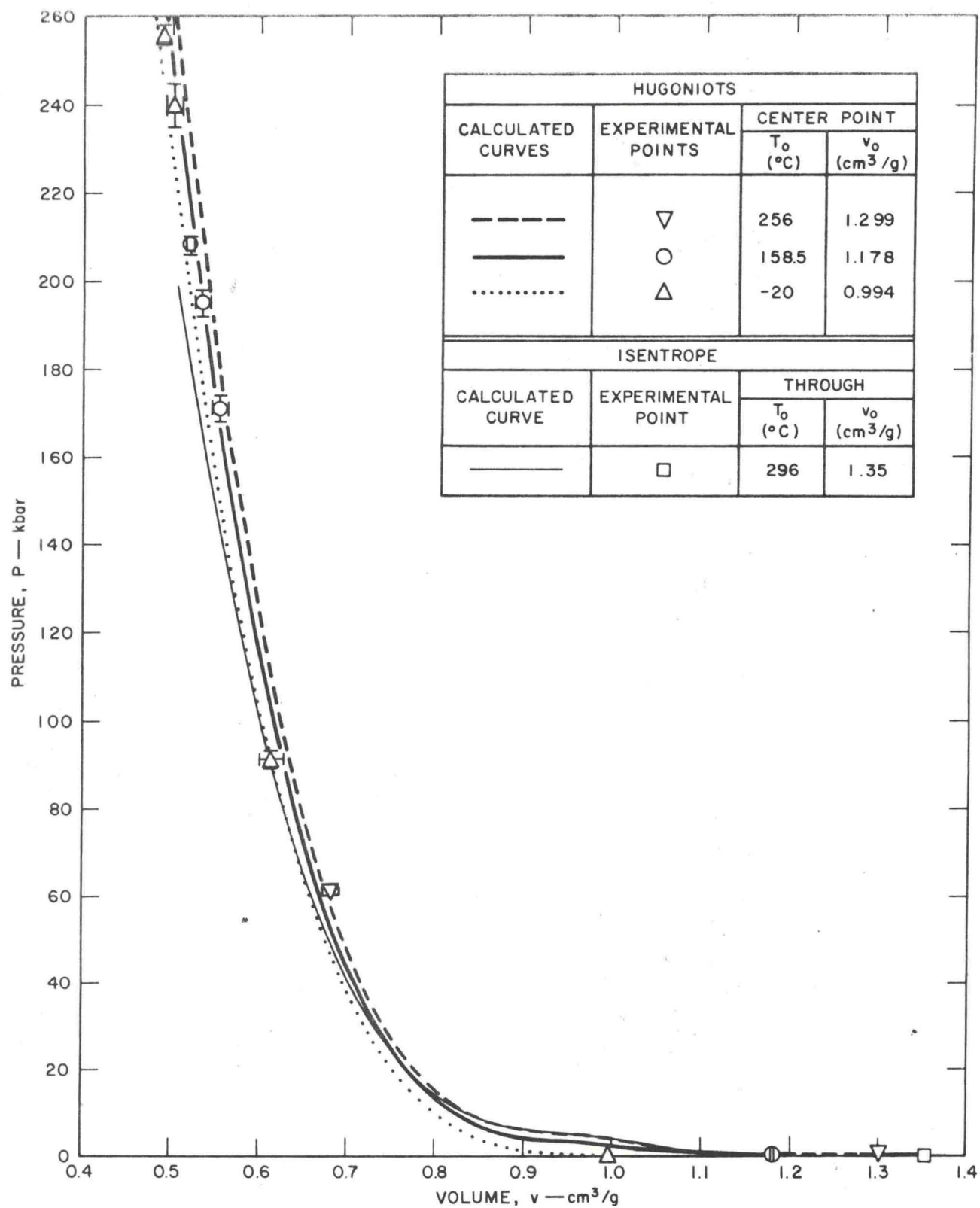


Fig. 3

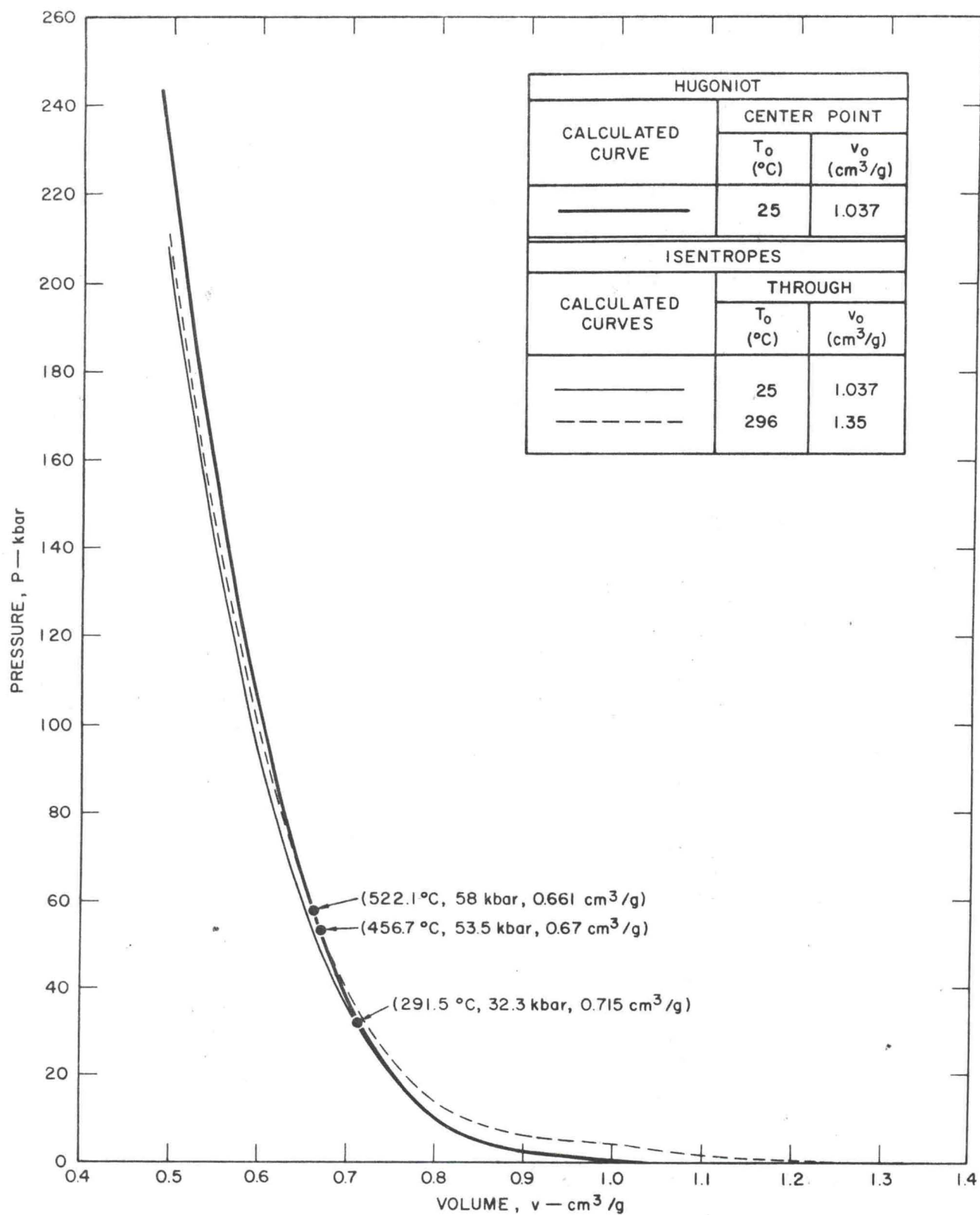
GP-4900-84





GC-4900-141

Fig. 4



GC-4900-140

Fig. 5

Table I  
SUMMARY OF SHOCK-WAVE EXPERIMENTS

SRI Shot No.	Initial Conditions			Shock Measurements		Calculated Shocked States		
	Initial Temp. (°C)	Specific Volume (cc/g)	Specific Energies (joules/g; ref. 0 at 25°C)	Shock Velocity <sup>a</sup> (mm/μsec)	Particle Velocity (mm/μsec)	Pressure <sup>b</sup> (kbar)	Specific Volume <sup>b</sup> (cc/g)	Specific Energy <sup>b</sup> (joules/g)
12,326	-32	0.983	-92	6.72 ± 0.02 <sup>c</sup>	3.18 ± 0.02	216 ± 2 <sup>c</sup>	0.518 ± 0.005 <sup>c</sup>	4952 ± 70 <sup>c</sup>
11,939	-20	0.994	-71	6.91 ± 0.07	3.46 ± 0.03	240 ± 5	0.497 ± 0.009	5887 ± 84
12,228	-20	0.994	-71	4.87 ± 0.05	1.87 ± 0.03	91 ± 2	0.611 ± 0.014	1684 ± 44
12,496	-10	1.004	-55	7.11 ± 0.04	3.11 ± 0.01	256 ± 2	0.494 ± 0.004	6474 ± 39
11,939	158	1.178	218	6.47 ± 0.06	3.56 ± 0.02	195 ± 3	0.530 ± 0.008	6548 ± 89
12,496	158	1.178	218	6.61 ± 0.02	3.72 ± 0.01	208 ± 2	0.516 ± 0.004	7225 ± 55
12,326	159	1.179	220	6.16 ± 0.04	3.28 ± 0.03	171 ± 3	0.550 ± 0.008	5607 ± 160
12,228	256	1.299	399	4.08 ± 0.02	1.94 ± 0.01	61 ± 1	0.679 ± 0.008	2290 ± 30
12,326	279	1.330	445	6.09 ± 0.07	3.31 ± 0.01	152 ± 2 <sup>c</sup>	0.606 ± 0.009 <sup>c</sup>	5927 ± 40 <sup>c</sup>
12,496	296	1.350	481	6.39 ± 0.05	3.91 ± 0.02	184 ± 3 <sup>c</sup>	0.526 ± 0.010 <sup>c</sup>	8095 ± 74 <sup>c</sup>

- a. Errors in shock velocity calculated by summing estimated errors in length and time measurements.
- b. Errors computed with absolute values of estimated errors; see ref. 9.
- c. Points omitted from analysis.



Table II  
CALCULATED TEMPERATURE ALONG THE  
25°C AND 296°C ISENTROPES

Volume (cm <sup>3</sup> /g)	25°C Isentrope (°C)	296°C Isentrope (°C)
1.35		296.0
1.29		300.4
1.25		304.0
1.19		310.8
1.15		316.4
1.09		328.5
1.05		339.7
1.0369	25.0	
1.01	29.4	353.9
0.969	38.0	371.6
0.909	50.8	398.3
0.849	64.3	426.1
0.809	73.6	445.4
0.749	88.0	475.4
0.709	98.1	496.2
0.649	113.7	528.5
0.609	124.5	551.0
0.549	141.3	585.9
0.509	153.1	610.1

Table III  
CALCULATED TEMPERATURES ON THE 25°C HUGONIOT ABOVE 58 KILOBAR

Pressure (kbar)	Volume (cm <sup>3</sup> /g)	Calculated with	
		$C_v = 1.9353 \times 10^{-2}$ kbar cm <sup>3</sup> /g°C $f(v) = 1.3822 + 0.10798v$	$C_v = 1.3735 \times 10^{-2}$ kbar cm <sup>3</sup> /g°C $f(v) = 1.7839$
		Temperature (°C)	Temperature (°C)
58.3	0.661	522.1	559.8
71.1	0.641	541.4	698.8
85.7	0.621	588.9	866.8
102.5	0.601	640.3	1069.8
121.5	0.581	725.4	1313.8
142.9	0.561	839.3	1591.8
167.0	0.541	986.4	1926.8
193.9	0.521	1171.3	2316.8
223.8	0.501	1397.3	2756.8